

## Chapter 12 Selection from Independent Projects under Budget Limitation

- **Basic Problem (Capital Budgeting Problem)**

- An investor can choose from a set  $S$  of  $n$  independent projects.
- At year 0, the investor has only  $b$  dollars to invest.
- Suppose that each project  $j \in S$  requires an initial investment of  $NCF_{j0}$ .
- Let  $NCF_{jt}$  be the net cash flows of project  $j$  at year  $t$ .
- Suppose that the investor can provide any capital needed beyond year 0.
- The problem is to select the subset of projects,  $A \subset S$  that maximizes the investor's fortune while meeting the budget constraint at year 0.

- **Solution Approach**

1. Identify “feasible bundles” that satisfy the budget constraint.

That is, find all  $A \subset S$ , such that  $\sum_{j \in A} NCF_{j0} \leq b$ .

2. Find the PW of each of these “feasible” bundles. The PW of a bundle is the sum of PWs of projects within the bundle.

That is,  $PW_A = \sum_{j \in A} PW_j$ .

3. Select the feasible bundle that gives maximum PW. That is, select  $A^*$  such that  $PW_{A^*} = \max_{A \in S} PW_A$  .

- **Underlying Assumption**

- PW of each project is determined on its own life span. No LCM or equal life approach is needed.
- This is equivalent to assuming that positive cash flows of short life projects in a bundle are reinvested throughout the life of the longest life project at the MARR.

- **Solving the capital budgeting problem using ILP**

- Solving the capital budgeting problem by hand can be tedious especially when the choice set  $S$  has many projects ( $2^n$  bundles should be considered).
- Integer linear programming (ILP) is an optimization technique that can be used to solve this problem quickly using a computer.
- In ILP, each project  $j \in S$  is assigned a binary variable  $x_j$ . The variable  $x_j$  can take on two values 0 and 1.
- If  $x_j = 0$ , the project is not selected. If  $x_j = 1$ , the project is selected.

- The ILP “model” for the capital budgeting problem is as follows:

$$\begin{aligned} \max \quad & Z = \sum_{j \in S} PW_j x_j \\ \text{subject to} \quad & \sum_{j \in S} NCF_{j0} x_j \leq b . \end{aligned}$$

- The ILP problem can be solved using several commercially available software (e.g., Microsoft Excel Solver).
- The solution to the ILP is denoted by  $\mathbf{x}^* = (x_1^*, x_2^*, \dots, x_n^*)$ . A project  $j$  with  $x_j^* = 1$  is selected. Otherwise,  $j$  is not selected.
- **Approximate (Heuristic) Capital Budgeting Solutions**
  - Approximate solutions to the capital budgeting ILP is obtained by ranking projects according to *benefit-cost ratio*.
  - This approximation method assumes that each project involves an initial outlay of cash (first cost) followed by a series of benefits.
  - The benefit-cost ratio for each project is the ratio of the *present value of benefits* to the initial cost.
  - The approximate solution is obtained by selecting the project with the highest benefit-cost ratio, then the project with the second highest ratio, and so on, until the budget is exhausted.